Fermi National Accelerator Laboratory

Possible Methods of Measuring the Length of Sub-Picosecond Electron Bunches in the Frequency Domain

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Possible Methods of Measuring the Length of Sub-Picosecond Electron Bunches in the Frequency Domain

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ABSTRACT

The traditional method of measuring extremely short electron beams using streak cameras begins to become problematic and expensive at bunch lengths at a picosecond and below. In this paper a few alternatives, based on the differential measurement of the Fourier spectrum of the longitudinal charge distribution of a bunch, are suggested and evaluated.

FOURIER TRANSFORM OF BUNCH PROFILES

Let us assume that the longitudinal beam current profile is Gaussian in shape and described by the equation (Q=Ne)

$$I_b(t) = \frac{Q}{\sqrt{2\pi}\sigma_\tau} e^{-\frac{t^2}{2\sigma_\tau^2}} \quad . \quad (1)$$

If Fourier transforms are defined by the equations

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t}\, dt \quad (2)$$

and

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{-i\omega t}\, d\omega \quad , \quad (3)$$

then the beam current in the frequency domain is described as

$$I_b(\omega) = Q\, e^{-\frac{1}{2}\sigma_\tau^2 \omega^2} \quad . \quad (4)$$

Therefore, in the frequency domain the power spectrum of the bunch shape is also Gaussian, but with an angular frequency width of $\sigma_\omega = 1/\sigma_\tau$. For M bunches each of charge Q_b constrained to occupy buckets of length T_o,

$$I_b(t) = \frac{Q_b}{\sqrt{2\pi}\sigma_\tau} \sum_{m=1}^{M} e^{-\frac{(t-mT_o)^2}{2\sigma_\tau^2}} \quad . \quad (5)$$

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The Fourier transform of (5) yields a frequency spectrum of

$$I_b(\omega) = \frac{Q_b}{\sqrt{2\pi}\sigma_\tau} \sum_{m=1}^{M} \int_{(m-\frac{1}{2})T_o}^{(m+\frac{1}{2})T_o} e^{-\frac{(t-mT_o)^2}{2\sigma_\tau^2}+i\omega t} \, dt \qquad , \qquad (6)$$

which can be reduced to

$$I_b(\omega) = Q_b \, e^{-\frac{1}{2}\sigma_\tau^2 \omega^2} \, Re\left\{ \Phi\left(\frac{T_o\sqrt{2}}{4\sigma_\tau} + i\omega \frac{\sigma_\tau}{\sqrt{2}} \right) \right\} \sum_{m=1}^{M} e^{i\omega m T_o} \qquad . \qquad (7)$$

The summation at the end of equation (7) is a form factor which describes the modulation of the frequency spectrum around harmonics of the bunch passage frequency. Figure 1 contains a plot of this form factor normalized to the number of bunches for M=1, 13, and 84 (cases commonly found at Fermilab).
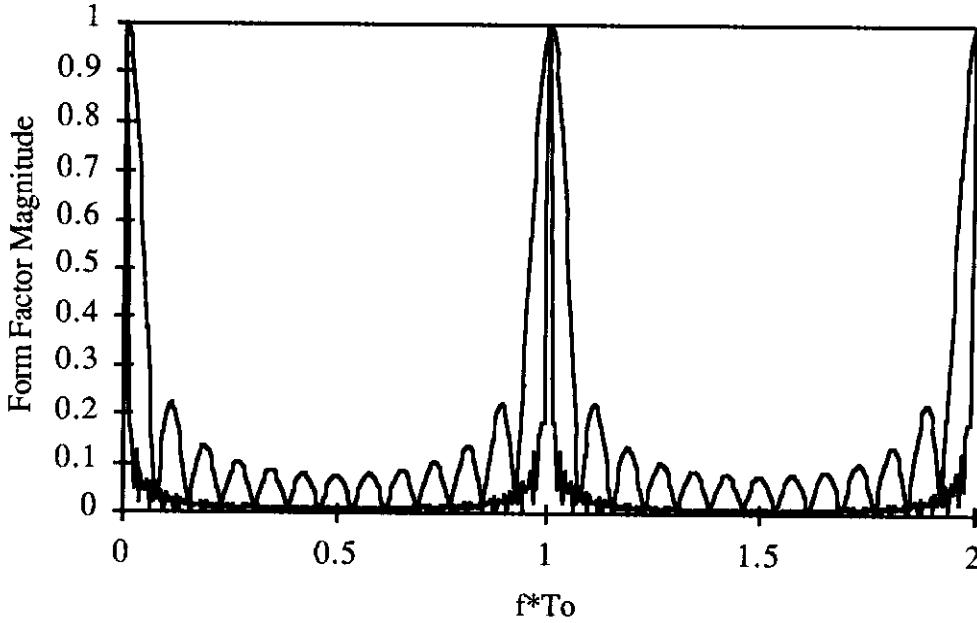

Figure 1: Sum of exp[iωmT$_o$] from m=1 to m=M (the total number of bunches) divided by the number of bunches. The top horizontal line is M=1, the wider modulated curve is M=13, and M=84 is the narrowest curve.

The shape of the curve described by equation (7) for M=1 is almost identical to that of equation (4) in the regime where the rms bunch length σ_τ is less than 1/6th the length of the RF bucket T_o. Figures 2 and 3 compare equations (4) and (7) at the bunch passage frequency and 3 times that frequency, respectively.
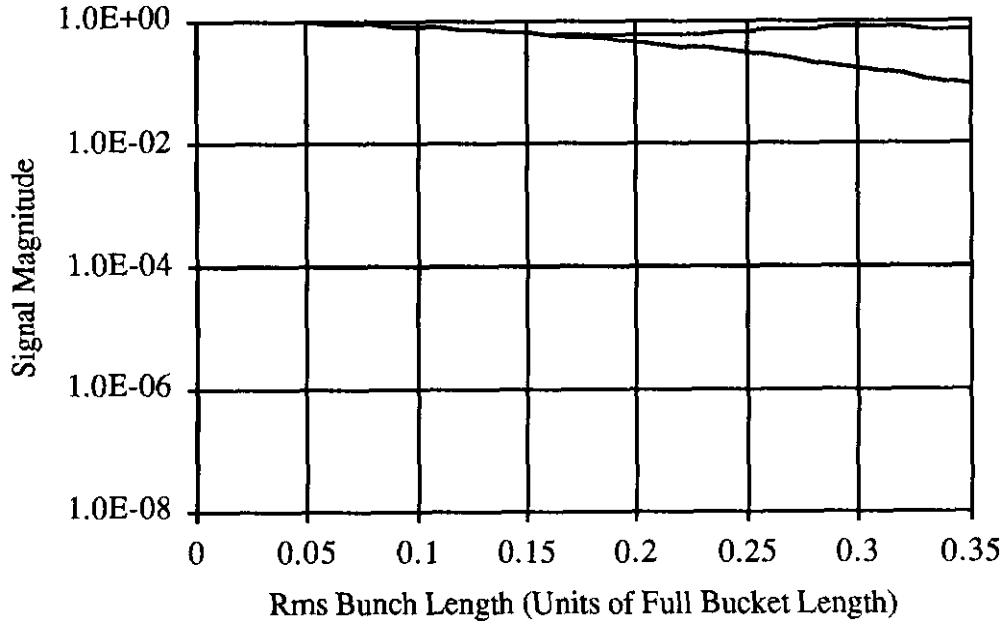
Figure 2: Spectral current density vs σ_τ/T_0 for $\omega=\omega_0$. The lower curve is from equation (4), and upper from equation (7).
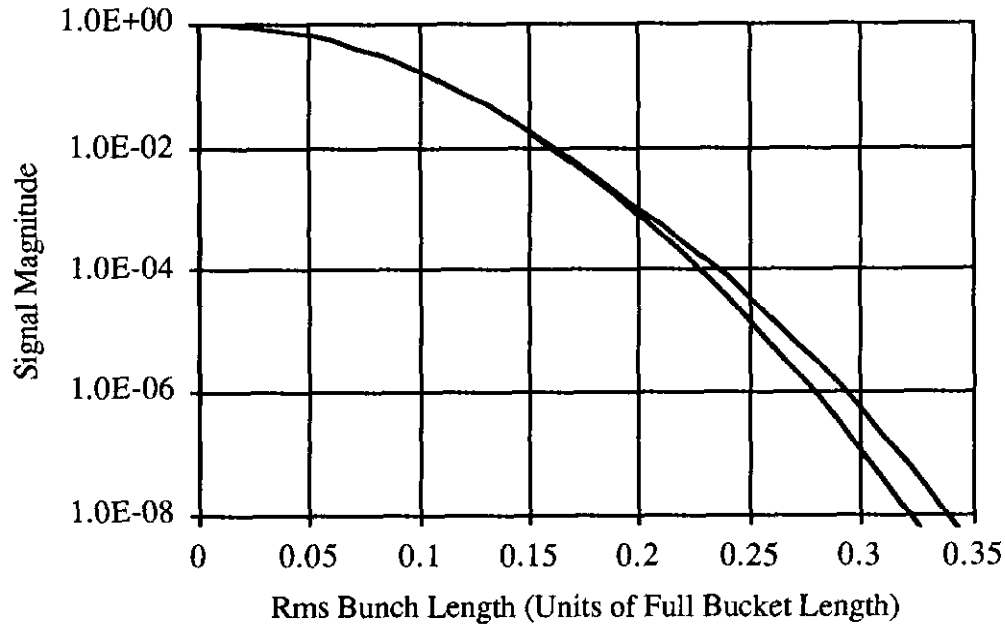

Figure 3: Spectral current density vs σ_τ/T_0 for $\omega=3\omega_0$. The lower curve is from equation (4), and upper from equation (7).

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REAL TIME BUNCH LENGTH MONITOR

In order to build a real time bunch length monitor, it is necessary to measure the value of the curve described by equation (4) at two different frequencies. The first frequency is the RF frequency, since the spectral current at this frequency (in units of DC beam current) is relatively insensitive to bunch length (see figure 2) and is independent of the number of bunches in the accelerator. The second frequency is chosen such that $\sigma_\tau \omega \approx 1$, so that the influence of tails is minimized while providing a measurable difference from the current at the lower frequency (see figure 3). Again, the exact frequency of the second line should be a harmonic of the RF frequency to insure that the spectral current (normalized to the DC current in the accelerator) is independent of the number of bunches.
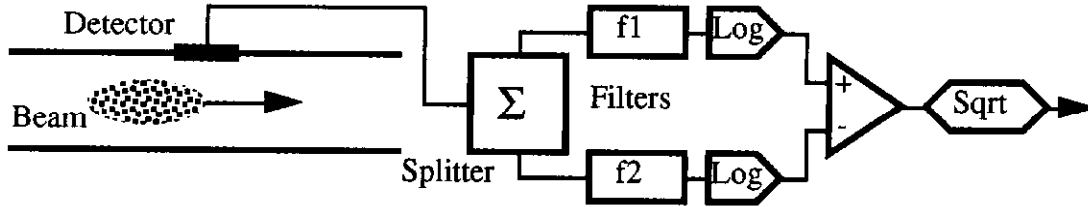

Figure 4: Schematic sketch of a standard real time bunch length monitor based on measuring the ratio of amplitudes at two frequencies of the bunch current spectrum.

As shown in figure 4, one uses a longitudinal, broadband beam current monitor of some type to measure the beam signal. Splitting this signal and filtering each half at the RF fundamental frequency and harmonic N of the RF frequency ω_0, the amplitude difference of two logarithmic detectors[1] yields an intermediate signal

$$S = Ln[I_b(\omega_0)] - Ln[I_b(N\omega_0)] \qquad ,$$

or

$$S = \left[-\tfrac{1}{2}\sigma_\tau^2\omega_0^2\right] - \left[-\tfrac{1}{2}N^2\sigma_\tau^2\omega_0^2\right] \qquad . \qquad (8)$$

Solving for σ_τ and performing an analog square root function using an Analog Devices AD532 semiconductor chip, the bunch length is described by the equation

$$\sigma_\tau = \frac{T_0}{2\pi} \sqrt{\frac{2}{(N^2 - 1)}} \sqrt{S} \qquad . \qquad (9)$$

Therefore, a real time (≈ 10 kHz analog bandwidth using the AD532) bunch length monitor insensitive to beam current and number of bunches has been created. Figure 5 is the result of applying equations (8) and (9) to the data in figures 2 and 3. All that is required is to measure two components of the beam spectral current. Monitors based on this principle are used in most of the Fermilab accelerators[2]. Despite the assumption that the beam profile is Gaussian, remarkably good agreement with the rms size of the actual waveform is generated.
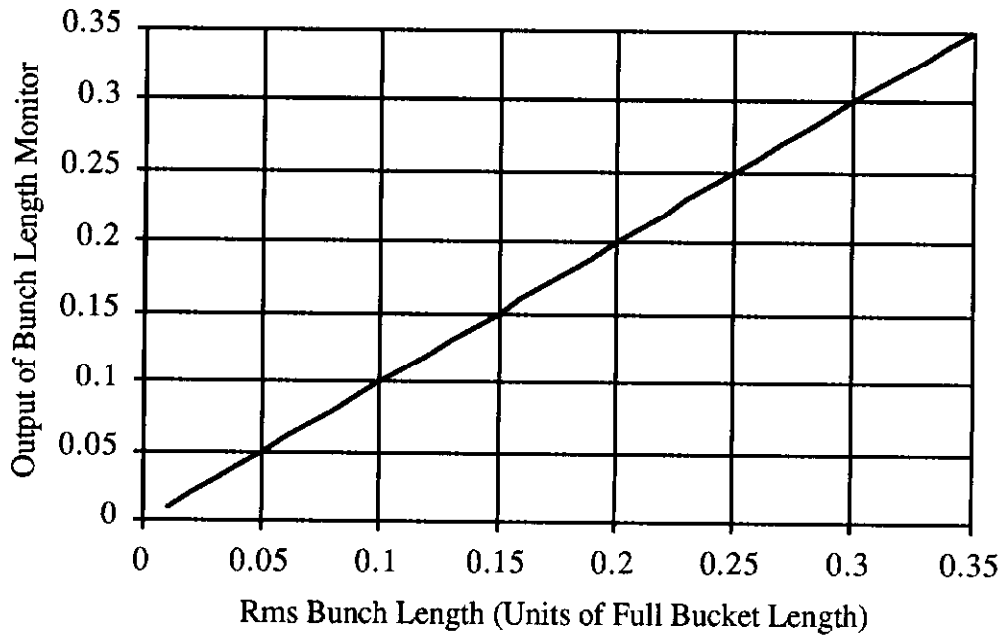
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Figure 5: Calculated bunch length vs. input length using equations (8) and (9). Actually, the signals applied to equations (8) and (9) were generated using both equations (4) and (7), and are both plotted (and are completely indistinguishable).

ELECTROMAGNETIC BUNCH LENGTH MONITOR

According to equation (4), to measure the length of a bunch which has a length of one picosecond or less, frequencies on the order of $1/2\pi\sigma_\tau \cong 200$ GHz must be detected. Since structures resonating at such high frequencies are so small, they must usually be attached to the wall of the beam pipe at a port through which electromagnetic energy can be coupled. Figure 6 contains a sketch of a possible geometry for a bunch length monitor.
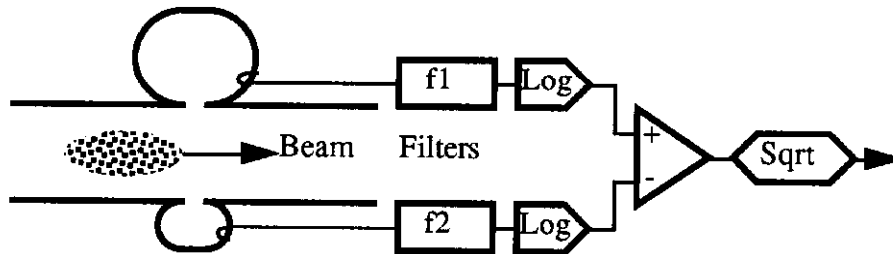

Figure 6: Sketch of a possible bunch length monitor in which two cavities with greatly different resonant frequencies are coupled to the image currents on the inside wall of the vacuum chamber via slots.

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The problem with this scheme when employed on bunches from electron guns is the distribution of electromagnetic fields from a non or mildly relativistic charge distribution. The approximate rms width of the longitudinal distribution of image charges on a perfectly conducting wall of radius R from a charge of relativistic energy γ and relativistic velocity β is

$$\sigma_w \cong \frac{R}{\gamma \beta c}$$

. (10)

Figure 7 contains a sketch of the electric field lines (and surface charge distribution) generated by a single particle. Because the beam pipe is assumed to be perfectly conducting, the longitudinal component of the electric field lines must be zero at the surface.
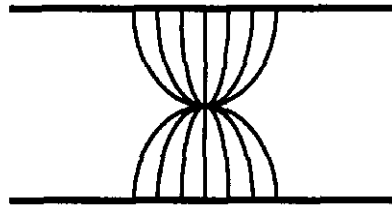

Figure 7: Field and image charge density distribution on the inside surface of an infinite conductivity beam pipe.
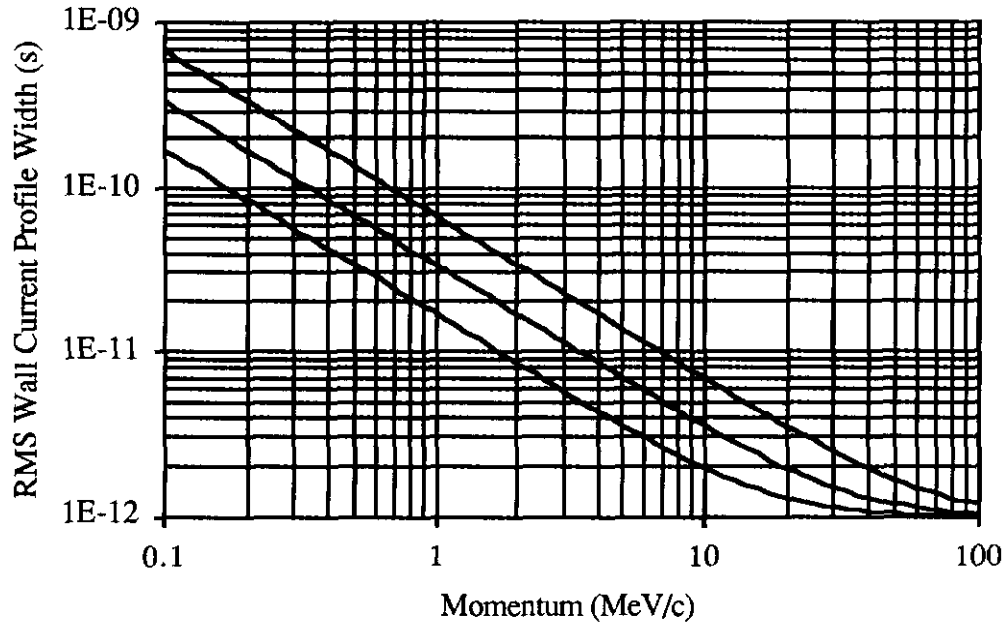

Figure 8: Effective bunch length measured at the beam pipe wall for a 1 psec bunch at different energies. The beam pipe radii were 4 cm (top), 2 cm (middle), and 1 cm (bottom curve).

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Using elementary kinematic relationships, the product of energy γ and velocity β can be rewritten as the beam momentum P_0 divided by the mass of the electron m_0. Convoluting over the actual bunch length, the effective bunch length measured by any detectors at the surface of the pipe is

$$\sigma_e = \sqrt{\sigma_\tau^2 + \left(\frac{m_0 R}{c P_0}\right)^2} \qquad . \qquad (11)$$

Figure 8 contains a plot of this effective signal width as a function of beam momentum assuming a 1 picosecond rms beam width and typical beam pipe radii of 1, 2 and 4 cm. Below a momentum of 10 MeV/c it becomes very difficult to correct the measurement value to recover the true beam width. Therefore, for RF guns whose output momentum is significantly less than 10 MeV/c, it is necessary to find some other means of measuring a spectral signature of the beam.

TRANSIENT SYNCHROTRON RADIATION SPECTRUM

In the body of a dipole magnet the spectrum of the synchrotron radiation is determined by the time the observer is illuminated by the Lorentz contracted light cone swinging by following the trajectory curvature of the magnet. In the case where the length of the magnet field is short in comparison with this "body" illumination time, frequency spreading of the synchrotron radiation spectrum occurs[3].
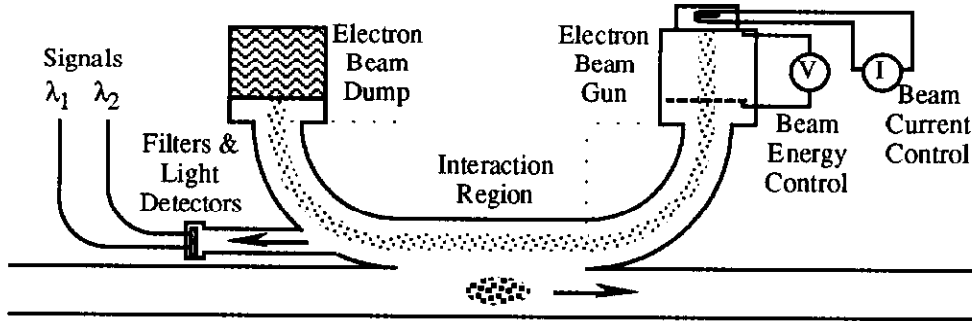

Figure 9: Proposed bunch length monitor using transient synchrotron radiation generated by the sharp beam-beam deflection of the picosecond bunch passing the low energy electron beam.

The transverse beam-beam deflection generated by a picosecond long bunch on a low energy electron beam can be described by an effective dipole field of peak magnitude B_0 and longitudinal position dependence

$$B(z) = B_0 \, e^{-\frac{z^2}{\sigma_\tau^2}} \qquad . \qquad (12)$$

The loss of the factor of two in the denominator of the exponential argument comes from the fact that the picosecond long bunch is moving and not a fixed magnet. The spectral density of the light from such a deflection is[4]

$$I(\lambda) = \frac{e^3}{2\pi^2\varepsilon_0 c} \frac{I_0}{2m_0^2 c^2}(\gamma B_0 c\sigma_\tau)^2 \exp\left[-\left(\frac{c\sigma_\tau}{2\gamma^2\lambda}\right)^2\right] \qquad . \qquad (13)$$

The argument of the exponential determines the range of energies required by the electron beam in the monitor to generate light in the wavelength band of available detectors (1-10 μm). Figure 10 is a plot of the energy of the monitor beam required to make the argument of the exponential in equation (13) unity as a function of the length of the picosecond long bunch under test. For sub-picosecond applications a very straightforward gun with an output energy of 5 MeV or less could be used for this monitor. The only outstanding questions are technical: Should one use head-on beam-beam collisions or impact parameters of 2σ of the test bunch? What detector technology is required to measure the fluxes from such a monitor?
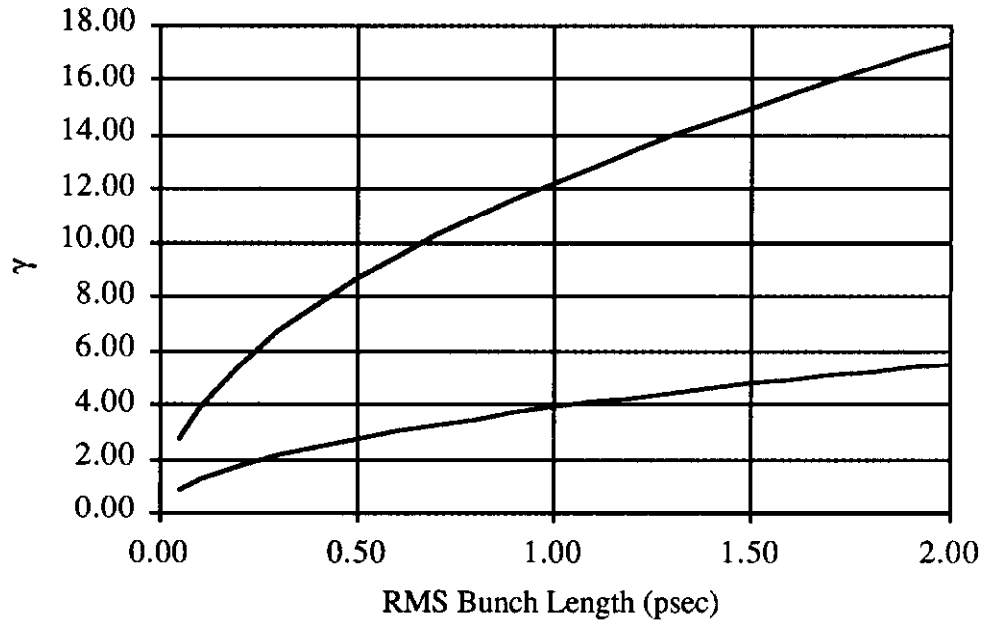

Figure 10: Relativistic energy γ needed to make the argument of the exponential in equation (13) unity for observation wavelengths of 1 μm (top) and 10 μm (bottom curve). The top curve would suggest that a monitor beam of 10 MeV is sufficient to create a useful bunch length detector.

OPTICAL TRANSITION RADIATION

When a relativistic beam traverses a thin foil, optical transition radiation[5] is produced. The frequency spectrum of the forward and backward propagating radiation is identical to that of the beam current[6]. Coupling the electromagnetic radiation from this transition through the metal surface of the foil into two resonant detectors at 200 GHz and 20 GHz (for example), one could construct the same bunch length monitor shown in figure 6 but without the pulse spreading phenomena caused by the low energy nature of some of the beams which need to be measured.

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CONCLUSIONS

A variety of techniques have been reviewed for the measurement of picosecond bunch lengths. They have all been based on the highly successful monitors used at Fermilab which provide real time bunch length information. Though they do not provide full pulse shape waveform and the answer can have some other meaning (FWHM, 2 sigma, etc...) for other nonGaussian beam shapes, the availability of a single real time "figure of merit" called the bunch length is a real boon to those tuning and studying accelerator performance.

ACKNOWLEDGMENTS

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